Note on the number of Faint Stars with large Proper Motions.

By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The existence of stars like Groombridge 1830, which is of 6·5 magnitude and has a P.M. of 700″ a century, larger than that of the brightest star known, suggests that there may be stars of (say) the 12th magnitude with proper motions so large as to be easily measurable. At first sight it would also appear that since there are many more stars of the 12th magnitude than of the 6th, the cases of large P.M. would also be more numerous. Accordingly, when Mr. C. L. Brook generously presented a stereocomparator to this observatory, search was made on plates for the Astrographic Catalogue which had, for one reason or another, been repeated after an interval of some years, for cases of sensible proper motion among faint stars. The results were most disappointing, practically nothing being found after a good deal of time spent in searching. It was thought that perhaps the réseaux on the two plates troubled the observer too much, and rendered his eyes less sensitive to small differences (see remarks by Dr. Max Wolf in *Mon. Not.*, lxiv. p. 112), and it was determined to take some plates without a réseau when the work of revision and printing of results for the catalogue was completed. The increase of the interval in the meantime would be a gain.

2. A recent visit of Professor Kapteyn to Oxford recalled attention to the question of P.Ms. of faint stars. He was anxious to get some provisional results, and it occurred to me that it would be worth while to compare the actual measures of plates of the same region taken at an interval of ten to fifteen years. We have many plates which were taken and measured in 1892 and following years, but which do not come up to the standard of the later work, and were accordingly repeated recently in the general revision which Mr. Bellamy has assiduously conducted as we come to print each zone. For stars in the same region, we thus have two sets of measures separated by an interval comparable with that proposed by Professor Kapteyn in his scheme for "sampling" the heavens; and since provisional constants have been deduced for each plate, the work of reducing one to the other is straightforward. It involves, of course, a considerable amount of arithmetical work; but it seemed desirable to face this labour in order to get a general idea of the results to be expected.

3. A number of regions have been already examined, and it soon became clear why we had drawn blank with the stereocomparator. The number of sensible P.Ms. of faint stars is in fact very small, and the results for +28° zone, which is now completed, will illustrate this point.

4. In zone +28° we had the following regions, giving an interval of over ten years.

R.A. of Centre.	Total Stars.	Interval.	Stars examined.	Reviser.	Centennial P.Ms. 15″–20″	>20″
h m		y				
0 36	182	12·2	6	J.H.W.	0	1
5 51	310	11·9	23	G.H.H.	2	0
7 30	191	12·1	14	H.H.T.	1	0
20 51	431	14·8	30	J.H.W.	1	0
21 0	359	14·8	20	H.H.T.	1	0
21 36	294	14·8	24	H.H.T.	0	2
21 45	264	14·8	11	J.H.W.	3	1
22 21	308	14·8	11	J.H.W.	0	1
22 30	230	14·8	10	G.H.H.	1	1
22 39	253	14·8	11	H.H.T.	1	0
Total	2822		160		10	6

Column 1 is the R.A. of plate centre, the declination being +28°.

Column 2 gives the total number of stars measured on the earlier plate, which was, from the nature of the case, deficient in stars, or it would not have been repeated. The earlier plate was reduced to the later by use of the constants obtained independently for each, which can be readily combined to give relative constants.

Column 3 gives the interval in years and tenths between the two exposures to the region.

Column 4 shows the number of cases of differences sufficiently large, either in x or y, to be worth examining specially. As a rule, all differences greater than $\pm 1''\cdot 3$ in x or y were examined. A large number of them were sensibly reduced on remeasurement, as was only to be expected; for they include all the cases where an extreme error of measurement in one direction on one plate happens to coincide with an extreme error in the opposite direction on the other plate. They also include cases of faulty images in the corners of the plates; of large elongated images of bright stars difficult to measure; and finally, a few (very few, we were glad to find) actual mistakes in the earlier plates,—none in the later plates, since such as existed had probably been caught in the proof-reading.

Column 5 shows the observer who examined the differences of column 4. In this work I am glad to acknowledge gratefully the volunteer assistance of Mr. J. H. Worthington, F.R.A.S., and Mr. G. H. Hamilton. Their remeasures were made with great care, in reversed positions of the plate.

Columns 6 and 7 show the cases of large P.M. detected. There were, of course, a number of smaller P.Ms. than these which are reasonably certain, viz.—

8	of centennial	P.M.	14″·0 to 14″·9
6	,,	,,	13·0 to 13·9
5	,,	,,	12·0 to 12·9
10	,,	,,	11·0 to 11·9
14	,,	,,	10·0 to 10·9

and so on. But these numbers cannot be considered accurate, since they would no doubt be increased by the addition of cases where a coincidence of errors in measurement annulled an existing P.M. Such cases could only be detected by re-measurement of all the stars, and this cannot be undertaken just now. For the moment, we are concerned with definite results only for the P.Ms. outside a certain limit, which has been provisionally fixed as 15″ centennial.

5. Now these large P.Ms. are by no means all of faint stars. They include several already well-known P.Ms., as for instance that of ϵ Andromedæ (Oxford +28° 1540_*). The asterisks in the second column of Table II. show that the corresponding stars are sufficiently bright to be in the Cambridge Meridian Catalogue, and it will be seen that there are 7 of these out of the 16. Table II. shows the details.

TABLE II.

Centennial P.M.	Oxford No.	Camb. No.	Δx.	Δy.	Mag.
″ 42·2	+28°·65996	...	+21	− 2	10·4
31·2	+28·66265*	12835	+10	−12	4·5
29·4	+28·69159	...	+14	+ 5	10·7
24·4	+28·67189*	12951	+10	+ 7	8·9
22·9	+28·69875	...	− 5	−10	10·8
22·8	+28·1540*	370	− 6	− 7	4·1
19·7	+28·61985*	11904	+ 8	− 6	8·0
18·9	+28·66897*	12981	− 8	− 5	6·8
17·1	+28·12579	...	+ 4	− 6	11·4
17·1	+28·66910	...	− 8	− 3	11·5
16·8	+28·69669	...	+ 8	+ 3	11·0
16·5	+28·63432	...	− 2	− 8	11·1
16·2	+28·70256*	13650	+ 8	− 1	9·0
16·1	+28·67027*	12959	− 7	− 4	9·3
15·9	+28·22900	...	+ 6	+ 2	10·9
15·4	+28·12121	...	+ 6	+ 2	11·2

6. Column 1 of Table II. shows the total P.M. per century.

Column 2 gives the Oxford number in vol. iv. of the Oxford Astrographic Catalogue; the asterisk denoting a reference star in the Cambridge A.G. Catalogue for 1875·0, the Cambridge number being given in Column 3.

Columns 4 and 5 show the total measured difference on the two plates in units of ·001 of a réseau space, or 0″·3. The intervals are smaller for some plates than others, which must be remembered in comparing this column with the first.

The last column gives a provisional magnitude, which for the bright stars is the Cambridge magnitude, and for the others is deduced from the formula printed at the head of each plate in the Oxford volume. These magnitudes are uncertain for the fainter stars, and may be a whole magnitude in error perhaps; but this will not affect the main point of the present note.

7. As a contrast with this table, we may take the results of the

new reduction of Groombridge's Catalogue recently published by the Royal Observatory at Greenwich. On p. cxi. of the Introduction is given a list of 9 stars with "extremely great proper motion," and another list of 163 stars with centennial P.M. greater than 20″, making 172 stars in all with P.M. greater than 20″ out of 4243. On the same scale, we should expect out of our 2822 stars, 115 with P.M. exceeding 20″, whereas we only get 6, and only 3 of these are of the faint stars which form the bulk of those examined. The discrepancy is very considerable, and even surprising. If we pick out the stars with P.M. greater than 20″ a century for the different magnitudes we get the following figures :—

Mag.	To 4·9.	5·0 to 5·9.	6·0 to 6·9.	7·0 to 7·9.	8·0 to 8·9.	Total.
No. Stars P.M. > 20″ .	34	35	49	35	19	172
Total Stars in Catalogue	234	538	1152	1374	930	4228
Percentage . . .	14·6	6·51	4·26	2·56	(2·05)	
Log (per cent.) . .	1·16	0·81	0·63	0·41	(0·31)	

The drop in the log (percentage) is about 0·20 per magnitude: the numbers for the 8·0 to 8·9 magnitudes are bracketed, since the search is probably far from complete. Carrying on this drop to magnitudes 11·0 to 11·9, we should get—

	8·0 to 8·9.	9·0 to 9·9.	10·0 to 10·9.	11·0 to 11·9.	Total.
Log (per cent.). . . .	0·21	0·01	9·81	9·61	
Percentage	1·62	1·02	0·65	0·41	
Total Stars on Oxford plates	60	216	684	1840	2800
Product	1·0	2·2	4·5	7·5	15·2

The distribution of the stars among the higher magnitudes is only a rough estimate, but cannot affect the expectation seriously. We see that, on the hypothesis that the drop in the log (percentage) is carried on uniformly, we might have expected 15 stars of over 20″ cent. P.M. and of magnitude fainter than 8·0, whereas we only get 4. The drop must therefore be much more rapid unless our figures are quite exceptional.

8. The work is being extended to other zones, and further results will be given at an early date. Meanwhile, the following remark seems worth making. Professor Kapteyn is proposing to determine the average Proper Motions of faint stars from photographs of specified regions taken at an interval of about fifteen years. Will this interval be long enough? The suggestion of the above results (it is, of course, only a suggestion at present) is that the P.Ms. are so small that a much longer interval will be necessary to disentangle them from the unavoidable accidental errors.

Summary.

There seem to be strangely few proper motions of over 20″ a century among the faint stars.

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On the Diminution of Light in its passage through Interstellar Space. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The question whether light suffers any diminution in its passage to us from distant stars is by no means new, and has been hitherto answered provisionally in the negative. Thus Newcomb writes in his book, *The Stars: a Study of the Universe* (p. 231, edition of 1901):—

"The conclusion that the universe is finite rests upon the hypothesis that light is never lost in its passage to any distance, however great. This hypothesis is in accordance with our modern theories of physics, yet it cannot be regarded as an established fact for all space, even if true for the distances of the visible stars. . . . The hypothesis of a limited universe and no extinction of light, while not absolutely proved, must be regarded as the one to be accepted until further investigation shall prove its unsoundness."

2. Newcomb is here considering loss of light to the ether. But light may be lost in other ways; and it has been found that a very reasonable hypothesis of scattering of light by small particles in space will explain the following facts:—

(*a*) The ratios of the number of stars per magnitude tabulated by Kapteyn as far as the twelfth magnitude.

(*b*) The fact that when photographic exposure is prolonged in a ratio which ought to give stars fainter by *five* magnitudes, we only get *four* visual magnitudes. If there is scattering of light by small particles, it should be greater for photographic than for visual rays, according to Rayleigh's well-known law of λ^{-4}; and the puzzling failure of photographic exposure, which has been attributed provisionally to the behaviour of the plate, is thus explained as a celestial phenomenon.

(*c*) The ratios of the counts of stars for different exposures found at Greenwich, with a range from $20^{\text{sec.}}$ to $40^{\text{min.}}$

(*d*) The well-known fact that the total light of all the stars is comparatively small.

So far as I am aware, these are *all* the available facts of a general nature; and a simple hypothesis which accounts for them all is, therefore, worth attention.

3. First consider the simple case when all the stars are of the same brilliance, B, and are uniformly distributed through space. Let the distance of a star be r, and its apparent magnitude m. Then—

$$10^{-m \times 0 \cdot 4} = \mathrm{B} r^{-2} \quad . \quad . \quad . \quad (1)$$

$$\therefore \quad dm \times 0 \cdot 4 \log_e 10 = 2\, dr/r \quad . \quad . \quad . \quad (2)$$